Nucl. Sci. and Tech., Vol.14, No. 2 (2024), pp. 11-15

Nuclear Science and Technology

Journal homepage: http://jnst.vn/index.php/nst

Unfolding method for surface activity density map reconstruction from ambient dose equivalent rate measurements based on solution of Fredholm equation of the 1st kind

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Abstract: A mathematical method has been developed for determining surface activity density maps from ambient dose equivalent rate measurements on a site with buildings, taking into account the shielding effect of the buildings by using a visibility matrix. The relationship between surface activity density and ambient dose equivalent rate is described through the Fredholm equation of the 1st kind and is numerically solved with a Tikhonov regularization. Use of the visibility matrix and raster cells made it possible to solve the Fredholm equations in barrier geometry to restore the density of the surface radionuclide contamination based on the ADER measurement. Method was used to restore locations of contamination with ^{137}Cs and its activities for the Andreeva Bay nuclear legacy site. The proposed method can be applied in the process of decontamination of radioactively contaminated territories, in particular during the remediation of the Andreeva Bay.

Keywords: *Fredholm equation; radionuclide surface contamination; nuclear legacy site.*

I. INTRODUCTION

Nuclear legacy sites are characterized by incomplete information about the radiation situation [1]. Radiation monitoring on nuclear legacy sites includes the measurement of gamma ambient dose equivalent rate (ADER) in workplaces and on the site of the radiation facility, and the measurement of radiation contamination levels within the health protection zone and supervision area, radioactive contamination of surfaces of workshops and equipment. Each dosimetry measurement conducted by facility personnel results in an exposure of those personnel. Moreover, incomplete information about the radiation characteristics can lead to incorrect assessment of doses when planning radiation hazardous work.

Therefore, the task of restoring the radiation situation parameters based on the available data is relevant for these sites. Mathematical methods that allow obtaining additional information about the radiation situation according to the available data help in optimizing radiation-hazardous work and allow minimizing personnel exposure doses. This paper presents a method for reconstructing surface activity density (SAD) from ADER measurements [2].

https://doi.org/10.53747/nst.v14i2.418

Received 07 September 2023, accepted 14 June 2024

II. CONTENT

A. Research subjects and methodology

The ADER calculation based on SAD can be made using the common known equations [3], both for workshops [4] and for contaminated sites [5]. However, the inverse problem is not so trivial. The problems connected with the determination of the radiation environment on the basis of instrumental measurements of SAD can be solved by simulating a SAD based on the measured ADER. There are various approaches to solve this problem. Generally, method of conversion coefficients (MCC) is used, based on the transition to SAD from the air kerma rate [6] or from ADER [7] by the appropriate transition coefficients. However, MCC can be applied only under conditions of low gradient in SAD across the surface.

For a complex radiation contamination a more sophisticated approach is required. This study offers a method for quantifying SAD from ADER measurements, the solution is based on the numerical solution of the Fredholm equation of the 1st kind, which is applicable for an arbitrary distribution of the SAD [8]. Instrumental measurements associated with an uncertainty; therefore, the task of the SAD restoring according to the ADER at the industrial site in mathematics is characterized as an ill-posed task. Tikhonov's regularization method was applied. In this article, the screening effect of buildings is taken into account by introducing the visibility function $Vis(x,y,\acute{x},\acute{y})$, into the kernel of the Fredholm equation; such a function characterizes the visibility of a point with coordinates $(\acute{x},\acute{y})$ from a point with coordinates (x,y). Then, ADER and SAD will be connected by the following Fredholm equation of the 1st kind:

$$P(x,y,L) = W \cdot \int\int \frac{K_\gamma \cdot Vis(x,y,\acute{x},\acute{y}) \cdot A(\acute{x},\acute{y})}{L^2+(x-\acute{x})^2+(y-\acute{y})^2} d\acute{x} d\acute{y}, \quad (1)$$

Where $P(x,y,L)$ is the ADER at the height L (m) above the surface at the point of the space with coordinates (x,y) (m) on the surface from all point radiation sources from the area of radionuclide contamination. $Vis(x,y,\acute{x},\acute{y})$ is equal to 1, if the line that connecting points (x,y) and $(\acute{x},\acute{y})$ does not pass through any buildings, otherwise 0 (unitless); $A(\acute{x},\acute{y})$ is the surface activity of radiation sources localized in the neighborhood of the point with coordinates $(\acute{x},\acute{y})$ (Bq); W is a normalizing factor ($1.20 \cdot 10^{-18}$ Sv/aGy for ^{137}Cs); L (m) is a height above the surface for a point with coordinates (x,y), K_γ[(aGy·m^2)/(sBq)] is a kerma-constant of the radionuclide.

The integral Fredholm equation of the first kind in (1) is solved by numerical methods through discretizing the integral equation [9]. When discretizing, the contaminated area is divided into square sections (cells). The degree of discretizing is given by the number of such cells along the length (M) and the width (N). For each site the measured P values and the desired activity A were determined. The problem was solved for an array of P measurements at one height (L = 1 m from the ground surface) [2].

$$P(x_m,y_n) = W \cdot \sum_{j=1}^{M}\sum_{i=1}^{N} Q(x_m,y_n,\acute{x}_i,\acute{y}_j) \cdot \acute{x} \cdot \acute{y} \cdot A(\acute{x}_i,\acute{y}_j), \quad (2)$$

Where $P(x_m,y_n)$ – P value being measured in the cell (x_m,y_n); $Q(x_m,y_n,\acute{x}_i,\acute{y}_j)$ – ADER rate in the cell (x_m,y_n) from the point source located in the cell $(\acute{x}_i,\acute{y}_j)$; $A(\acute{x}_i,\acute{y}_j)$ – activity in the cell $(\acute{x}_i,\acute{y}_j)$; $\acute{x}$ and $\acute{y}$ – the cell width along axis X and Y, respectively.

The data of instrumental measurements always contain an error, therefore, in our case the system of linear algebraic equations (3) will be incorrect, and its solution requires an application of methods to solve ill-posed problems. In our case, the equation was solved by the Tikhonov's regularization method [10]. To solve the problem, assumption was made about the smoothness of the solution. And second assumption is that the SAD is not negative value. In this case, instead of the original system of equations (3), the modified equation was solved.

Assume that in equation (1), the P value and the surface activity A are discretized on the same grid with a constant step along the X and Y axes equal to h_x and h_y, respectively. The following notation is introduced:

$$Q(x_m, y_n, \acute{x}_i, \acute{y}_j) = q_{m,n,i,j} \quad (3)$$

$$A(\acute{x}_i, \acute{y}_j) = a_{i,j} \quad (4)$$

$$P(x_m, y_n) = p_{m,n} \quad (5)$$

Then, the system of equations (2) can be rewritten as follows:

$$p_{m,n} = W \cdot \sum_{j=1}^{M} \sum_{i=1}^{N} q_{m,n,i,j} \cdot h_x \cdot h_y \cdot a_{i,j} \quad (6)$$

or:

$$f_{m,n} = p_{m,n}/(h_y \cdot W) = \sum_{j=1}^{M} \sum_{i=1}^{N} q_{m,n,i,j} \cdot h_x \cdot a_{i,j} \quad (7)$$

Later, the matrix representation of q array is more suitable $\boldsymbol{Q} = \|q\|$:

$$q_{m+M\cdot(n-1),i+N\cdot(j-1)} = q_{m,n,i,j}, \quad (8)$$

as well as the vector representations of $\boldsymbol{F}$ and $\boldsymbol{a}$ matrix $\|f\|$ and $\|a\|$:

$$f_{m+N\cdot n} = f_{m,n} \quad (9)$$

$$a_{m+N\cdot n} = a_{m,n} \quad (10)$$

Publication [11] considers the system of equations (2) for the one-dimensional grid of $M\times N$ cell size with step h_x, kernel (core) $\boldsymbol{Q}$ and response function $\boldsymbol{F}$:

$$\boldsymbol{B}^{\alpha} \cdot \boldsymbol{a} = \boldsymbol{F} \quad (11)$$

$$\boldsymbol{F} = \{f_l\},\ l = 1, \dots, M \times N \quad (12)$$

$$\boldsymbol{B}^{\alpha} = \boldsymbol{Q} + \alpha \boldsymbol{C} \quad (13)$$

$$\boldsymbol{Q} = \{q_{ik}\}, \quad i = 1, \dots, M \times N; \quad k = 1, \dots, M \times N \quad (14)$$

$$\boldsymbol{C} = \boldsymbol{E} + \boldsymbol{C}_1 \quad (15)$$

$$\boldsymbol{C}_1 = \begin{pmatrix} \frac{1}{h_x^2} & -\frac{1}{h_x^2} & 0 & \cdots & 0 \\ -\frac{1}{h_x^2} & \frac{2}{h_x^2} & -\frac{1}{h_x^2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & -\frac{1}{h_x^2} \\ \cdots & \cdots & \cdots & -\frac{1}{h_x^2} & \frac{1}{h_x^2} \end{pmatrix} \quad (16)$$

It should be stressed that in this way an "effective" SAD is restored at a given point of the surface, which may differ from the instrumentally measured SAD at this point, because sources belonging to a large area contribute to the measured value P, regardless of their depth distribution.

B. Results

The SAD was calculated for the main dose-forming radionuclide ^{137}Cs using the ADER measurement array at a height of H = 1 m from the ground for a fragment of the industrial site of the Andreeva Bay (Northwest Center for Radioactive Waste Management (SevRAO) of the Federal State Unitary Enterprise 'Federal Environmental Operator'). Radionuclide contamination on this fragment occurred as a result of an accident: a leak of radioactive water from the spent nuclear fuel storage building [1]. The calculation results are shown in Fig. 1.

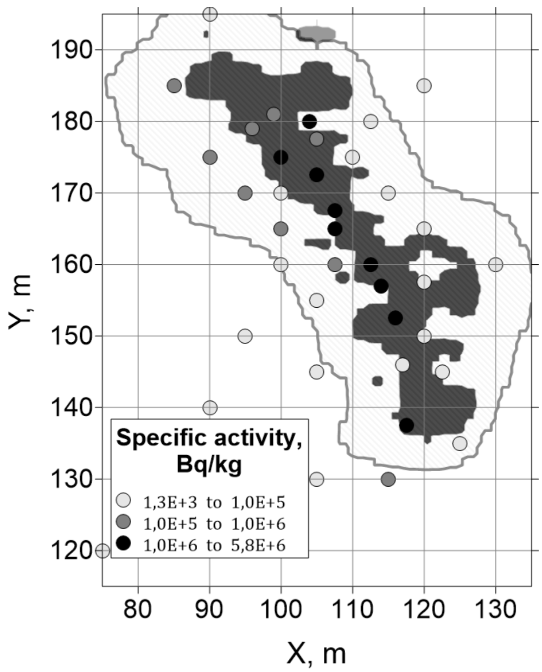

Fig. 1. Results of ^{137}Cs SAD recovery according to ADER in the studied fragment of the industrial site. Comparison results for ^{137}Cs for SAD and specific activity (gray and black squares)

Validation of the proposed method was carried out on the basis of 245 measurements of the specific activity of ^{137}Cs in soil samples at the industrial site in Andreeva Bay nuclear legacy site [1]. Figure 1 shows the results of measuring the specific activity of ^{137}Cs in soil samples in the studied fragment of the industrial site (black, dark gray and light gray squares), the results of calculating the SAD according to the MCC (area with light gray slashed background) and according to the proposed method (dark gray area). The information coefficient of correlation between the calculated and measured values was R = 0.68, which proves the adequacy of the method from the point of view of the real SAD.

C. Discussion

It has been found that the area of sites contaminated with ^{137}Cs, determined by the method proposed in this article, is 2-4 times less than the area determined by the MCC, which is characterized by "blurring" of the SAD due to the inherent smoothing of the MCC.

Consequently, the method developed gives a clearer idea of the area of contaminated sites, which can significantly reduce the amount of rehabilitation work. To solve an ill-posed problem, it is necessary to choose the regularization parameter α in eq. (24). It is selected manually, based on initial assumptions about the nature of surface contamination. After additional measurements, it is necessary to correct the model.

The method was developed for the situation when the radiation background is given by only one radionuclide. For several radionuclides, superposition should be used. The method does not provide information on the depth distribution of contamination, so we should talk about effective radionuclide surface contamination. The method is implemented as a computer code. User should take into account the technical capabilities of the computer, because for solving systems of linear equations, a large amount of RAM is required.

III. CONCLUSIONS

The map of the ^{137}Cs surface contamination being built on the basis of the measurement of the ambient dose equivalent rate through the numerical solving of the Fredholm equation of the first kind by Tikhonov's regularization method, allowed the rather accurate evaluation of the surface contamination density of the Andreeva Bay industrial site with ^{137}Cs radionuclide without increasing the amount of measurements. To clarify the compliance of calculated and measured values of the surface contamination density with ^{137}Cs, an informational correlation factor has been calculated. For soil in the vicinity of the creek flowing from under Building 5 at the Andreeva Bay industrial site this factor was R = 0.68. This confirms the average correlation of data and applicability of the method developed.

The proposed method is more general in comparison with MCC, because it is applicable for any distribution of the surface activity.

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